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Unsteady forced and natural convection around a sphere immersed in a porous medium

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Abstract. Unsteady forced and natural convection around a sphere immersed in a fluid-saturated porous medium is investigated. The sphere is suddenly heated and, subsequently, maintains a constant temperature over the surface. For the forced convection problem, the method of matched asymptotic expansions is used to obtain an asymptotic solution of the energy equation in terms of the Peclet number. For the natural convection problem, asymptotic solutions in terms of the Rayleigh number are obtained by means of a regular perturbation method.

Key words: Porous media, Unsteady convection, Sphere, Matched asymptotics

1. Introduction

Convection heat transfer around a body embedded in a fluid-saturated porous medium is a very important subject area, because of its wide applications in geophysics and engineering [1,2]. The present paper is concerned with both forced and natural convection around a sphere immersed in a porous medium. An example of an application of this problem is the environmental impact of buried nuclear heat-generating waste. Previous studies on forced convection around a sphere have been made by Sano [3] and Cheng [4]. In these studies, the authors considered the steady-state convection around an isothermal sphere and asymptotic solutions for small [3] and large [4] Peclet numbers were obtained, assuming a Darcy flow for the velocity field. The unsteady forced convection problem around a sphere, on the other hand, has not been considered so far. One of the purposes of the present paper is to analyze unsteady forced convection at low Peclet number around a sphere which is suddenly heated and, subsequently, maintains a constant temperature over the surface. The method of matched asymptotic expansions is used to obtain a solution of the energy equation. The related problem for the case of a circular cylinder involving an approach similar to that of the present paper has already been considered by Sano [5]. In the problem for a circular cylinder, the solution for the temperature is expressed as an expansion in inverse powers of log Pe, Pe being the Peclet number, while, in the present problem for a sphere, the solution is expressed as an expansion in terms of Pe, as we shall see later. This fact suggests that the sphere problem is more interesting in a physical sense than the circular cylinder problem, because the effect of convection appears in the energy equation, even in the inner region near the surface.

The second problem considered in this paper is natural convection around a sphere in a porous medium. Several authors [6, 7, 8, 9, 10] have already considered the steady natural convection around a sphere with isothermal or non-isothermal surface. Recently, Ganapathy and Purushothaman [11], Ganapathy [12] and Sano and Okihara [13] investigated unsteady natural convection around a sphere. Sano and Okihara obtained asymptotic solutions for small Rayleigh numbers for the case when the sphere is suddenly heated and, subsequently, maintains a constant heat flux over the surface. In the present paper, we extend this analysis to another important case when a sphere is suddenly heated and, subsequently, maintains a constant temperature at the surface.

2. Forced-convection problem

2.1. Governing equations

We first consider forced convection around a sphere immersed in a steady Darcy flow. It is assumed that the superficial velocity of the flow far upstream is uniform ($= U_\infty$) and that, initially, the surface of the sphere and the surroundings are at the same temperature T_∞, whereupon at time $\tau' = 0$ the surface temperature is suddenly changed to a constant value T_w. The energy equation governing the unsteady temperature field caused by this step change in wall temperature can be written in non-dimensional form as

$$\frac{\partial T}{\partial \tau} + \mathrm{Pe}\left(u_r \frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial \theta}\right) = \nabla^2 T \tag{1a}$$

$$u_r = (1 - r^{-3})\cos\theta, \quad u_\theta = -\frac{1}{2}(2 + r^{-3})\sin\theta, \tag{1b}$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial}{\partial \theta}\right), \tag{1c}$$

where non-dimensional quantities are defined as

$$r = r'/r_0, \quad \tau = \lambda_c \tau'/(r_0^2 \rho_c c_c), \tag{2a}$$

$$(u_r, u_\theta) = (u_r'/U_\infty, u_\theta'/U_\infty), \quad T = (T' - T_\infty)/(T_w - T_\infty), \tag{2b}$$

$$\alpha = \lambda_c/\rho_f c_f, \quad \mathrm{Pe} = U_\infty r_0/\alpha \quad \text{(Peclet number)}. \tag{2c}$$

In the above equations, T' is the locally averaged temperature, τ' the time, (r, θ, ϕ) denote spherical coordinates with $r' = 0$ at the center of the sphere and $\theta = 0$ in the direction of uniform flow, u_r' and u_θ' the superficial velocities in the r'- and θ-directions, respectively, r_0 the radius of the sphere, ρ_f and c_f the density and specific heat of the fluid, ρ_c, c_c and λ_c the density, specific heat and effective thermal conductivity of the saturated porous medium.

The non-dimensional initial and boundary conditions are

$$\tau < 0 \quad T = 0 \tag{3a}$$

$$\tau \geqq 0 \quad T = 1 \quad \text{at} \quad r = 1, \tag{3b}$$

$$T \to 0 \quad \text{as} \quad r \to \infty. \tag{3c}$$

2.2. Asymptotic solution for small Pe

We shall now proceed to obtain an asymptotic solution of the energy equation (1) for small Peclet number, using the method of matched asymptotic expansions. The matching procedure is similar to that used in papers by Sano [5, 14], where unsteady heat transfer from a circular cylinder in a low-Peclet-number Darcy flow [5] and unsteady low-Reynolds-number flow around a sphere [14] were analyzed.

In the small-time domain, where $\tau = O(1)$, the solution of (1) is expanded as

$$T = T_0(r, \theta, \tau) + \mathrm{Pe}\, T_1(r, \theta, \tau) + \cdots. \tag{4}$$

This expansion is uniformly valid for $1 \leqq r \leqq \infty$, since, in this domain, the temperature layer caused by the step change in surface temperature is confined to the region near the surface and the convective effects are negligible everywhere compared to the conduction and unsteady terms. Inserting (4) into (1a), we obtain equations for the T_n's as follows:

$$\frac{\partial T_0}{\partial \tau} - \nabla^2 T_0 = 0, \tag{5a}$$

$$\frac{\partial T_n}{\partial \tau} - \nabla^2 T_n = -\left(u_r \frac{\partial T_{n-1}}{\partial r} + \frac{u_\theta}{r}\frac{\partial T_{n-1}}{\partial \theta}\right) \quad \text{for} \quad n \geqq 1. \tag{5b}$$

The solutions for T_0 and T_1 satisfying the boundary conditions both on the surface and at infinity are, respectively.

$$T_0 = r^{-1} \operatorname{erfc} \eta, \tag{6}$$

$$\begin{aligned} T_1 = \; & \frac{1}{4}\Big\{3(r^{-2} - r^{-1}) \exp(r - 1 + \tau) \operatorname{erfc}(\eta + \tau^{1/2}) \\ & + (r^{-3} - 3r^{-2} + 2) \operatorname{erfc} \eta\Big\} \cos\theta, \end{aligned} \tag{7}$$

where

$$\eta = \frac{r - 1}{2\tau^{1/2}}. \tag{8}$$

On the other hand, the corresponding steady solution for T_1 was given by Sano [3] as

$$T_1 = -\frac{1}{2}(1 - r^{-1}) + \frac{1}{2}\left(1 - \frac{3}{2}r^{-2} + \frac{1}{2}r^{-3}\right)\cos\theta. \tag{9}$$

Apparently, (7) cannot approach this steady solution as $\tau \to \infty$, meaning that (7) and hence expansion (4) are invalid in the large-time domain, where $\tau = O(\mathrm{Pe}^{-2})$. This is due to the fact that, as τ becomes large, the temperature layer diffuses into the outer region, where $r = O(\mathrm{Pe}^{-1})$ and convection effects are not negligible. Therefore, the temperature field for large τ has a two-region structure in r, namely, large-time inner region and large-time outer region.

A time variable appropriate for large τ is

$$\tau^* = \mathrm{Pe}^2 \tau. \tag{10}$$

In the inner region near the surface, the energy equation may be written as

$$\mathrm{Pe}^2 \frac{\partial T^{(i)}}{\partial \tau^*} + \mathrm{Pe}\left(u_r \frac{\partial T^{(i)}}{\partial r} + \frac{u_\theta}{r}\frac{\partial T^{(i)}}{\partial \theta}\right) = \nabla^2 T^{(i)}, \tag{11}$$

where

$$T^{(i)}(r,\theta,\tau^*) = T(r,\theta,\tau) \tag{12}$$

The solution of (11) is assumed to be of the form (large-time inner expansion)

$$T^{(i)} = T_0^{(i)}(r,\theta,\tau^*) + \mathrm{Pe}\, T_1^{(i)}(r,\theta,\tau^*) + \cdots. \tag{13}$$

In the outer region far from the sphere, where $r = O(\mathrm{Pe}^{-1})$, we introduce the following outer variables

$$\rho = \mathrm{Pe}\, r, T^{(0)}(\rho,\theta,\tau^*) = \mathrm{Pe}^{-1} T^{(i)}(r,\theta,\tau^*), \tag{14}$$

in terms of which the energy equation becomes

$$\frac{\partial T^{(0)}}{\partial \tau^*} + (1 - \mathrm{Pe}^3\rho^{-3})\cos\theta \frac{\partial T^{(0)}}{\partial \rho} - \frac{1}{2}\rho^{-1}(2 + \mathrm{Pe}^3\rho^{-3})\sin\theta \frac{\partial T^{(0)}}{\partial \theta} = \nabla_\rho^2 T^{(0)}, \tag{15}$$

where ∇_ρ^2 is the same operator as ∇^2, but with r replaced by ρ. This equation reflects the proper balance between the convection and conduction terms in the outer region. The solution of (15) is assumed to be (large-time outer expansion)

$$T^{(0)} = T_0^{(0)}(\rho,\theta,\tau^*) + \mathrm{Pe}\, T_1^{(0)}(\rho,\theta,\tau^*) + \cdots. \tag{16}$$

The boundary conditions at the surface are imposed on the inner expansion (13) and the one at infinity on the outer expansion (16). The matching condition between (13) and (16) may be written as

$$\lim_{r\to\infty} T^{(i)} = \lim_{\rho\to 0} T^{(0)}. \tag{17}$$

Furthermore, the outer expansion is required to satisfy the matching condition with the small-time expansion (4), which may be written as

$$\lim_{\tau^*\to 0} T^{(0)} = 0. \tag{18}$$

This is because the thermal layer in the small-time domain is confined to the inner region near the surface.

The solutions are obtained for $T_0^{(i)}, T_1^{(i)}$ and $T_0^{(0)}$. Since the procedure for obtaining them is straightforward, only the final results will be shown below.

$$T_0^{(i)} = r^{-1}, \tag{19}$$

$$\begin{aligned} T_1^{(i)} = & -(1 - r^{-1})\left\{\frac{1}{\sqrt{\pi\tau^*}}\exp\left(-\frac{\tau^*}{4}\right) + \frac{1}{2}\operatorname{erf}\left(\frac{\sqrt{\tau^*}}{2}\right)\right\} \\ & + \frac{1}{2}\left(1 - \frac{3}{2}r^{-2} + \frac{1}{2}r^{-3}\right)\cos\theta, \end{aligned} \tag{20}$$

$$\begin{aligned} T_0^{(0)} = & \frac{1}{2\rho}\left\{\exp\left(\frac{\rho}{2}\right)\operatorname{erfc}\left(\frac{\rho}{2\sqrt{\tau^*}} + \frac{\sqrt{\tau^*}}{2}\right)\right. \\ & \left. + \exp\left(-\frac{\rho}{2}\right)\operatorname{erfc}\left(\frac{\rho}{2\sqrt{\tau^*}} - \frac{\sqrt{\tau^*}}{2}\right)\right\}\exp\left(\frac{1}{2}\rho\cos\theta\right). \end{aligned} \tag{21}$$

It can easily be shown that, as $\tau^* \to \infty$, these solutions approach the corresponding steady solutions given in [3].

2.3. Discussions

Fig. 1 shows the isothermal lines for Pe $= 0.2$ calculated from the solutions obtained so far. The result for $\tau = 1$ is calculated from (6) and (7) and those for $\tau = 25$ ($\tau^* = 1$) and $\tau \to \infty$ from the inner solutions (19) and (20). It is seen that the isotherms grow in the downstream direction, which may be attributed to convection. Since the convective effect is larger for larger values of r, the growing is more remarkable for larger values of r.

The local Nusselt number defined by $\mathrm{Nu} = hr_0/\lambda_c$, where $h = -(\lambda_c/(T_w - T_\infty))(\partial T'/\partial r')_{r'=r_0}$ is the heat transfer coefficient, may be calculated from (6) and (7) as

$$\mathrm{Nu} = 1 + \frac{1}{\sqrt{\pi\tau}} - \frac{3}{4}\mathrm{Pe}\{1 - \exp(\tau)\mathrm{erfc}(\tau^{1/2})\}\cos\theta + O(\mathrm{Pe}^2), \tag{22}$$

for small time ($\tau = O(1)$) and from (19) and (20) as

$$\mathrm{Nu} = 1 = \mathrm{Pe}\left\{\frac{1}{\sqrt{\pi\tau^*}}\exp(-\tau^*/4) + \frac{1}{2}\mathrm{erf}(\tau^{*1/2}/2) - \frac{3}{4}\cos\theta\right\} + O(\mathrm{Pe}^2), \tag{23}$$

for large time ($\tau = O(\mathrm{Pe}^{-2})$). From (22) and (23), we can construct a single composite expansion for Nu which is valid for all values of time as

$$\begin{aligned}\mathrm{Nu} &= 1 + \mathrm{Pe}\left[\frac{1}{\sqrt{\pi\tau^*}}\exp(-\tau^*/4) + \frac{1}{2}\mathrm{erf}(\tau^{*1/2}/2)\right.\\ &\quad \left. + \frac{3}{4}\{-1 + \exp(\mathrm{Pe}^{-2}\tau^*)\,\mathrm{erfc}\,(\mathrm{Pe}^{-1}\tau^{*1/2})\}\cos\theta\right] + O(\mathrm{Pe}^2).\end{aligned} \tag{24}$$

From this, the mean Nusselt number $\overline{Nu}$ averaged over the surface of the sphere may be calculated as

$$\begin{aligned}\overline{\mathrm{Nu}} &= \frac{1}{2}\int_0^\pi Nu\sin\theta\,\mathrm{d}\theta = 1 + \mathrm{Pe}\left\{\frac{1}{\sqrt{\pi\tau^*}}\exp(-\tau^*/4) + \frac{1}{2}\mathrm{erf}(\tau^{*1/2}/2)\right\}\\ &\quad + O(\mathrm{Pe}^2).\end{aligned} \tag{25}$$

Fig. 2 shows the relation between $\overline{\mathrm{Nu}}$ and τ for several values of Pe. The result for Pe $= 0$ corresponds to the conduction solution. It is seen that $\overline{\mathrm{Nu}}$ decreases monotonically to its steady value as τ increases and that the response time of heat transfer is smaller for larger values of Pe. Furthermore, we can see that for small time ($\tau = O(1)$), $\overline{\mathrm{Nu}}$ does not depend on the value of Pe, meaning that convection does not influence the mean Nusselt number at small time. This fact can also be understood from (22): The convection term (second term) in (22) is proportional to $\cos\theta$ and contributes nothing to $\overline{\mathrm{Nu}}$.

3. Natural convection problem

3.1. Governing Equations

Now we shall consider natural convection around a sphere immersed in a fluid-saturated porous medium. It is assumed that, as in the previous problem, the surface temperature of

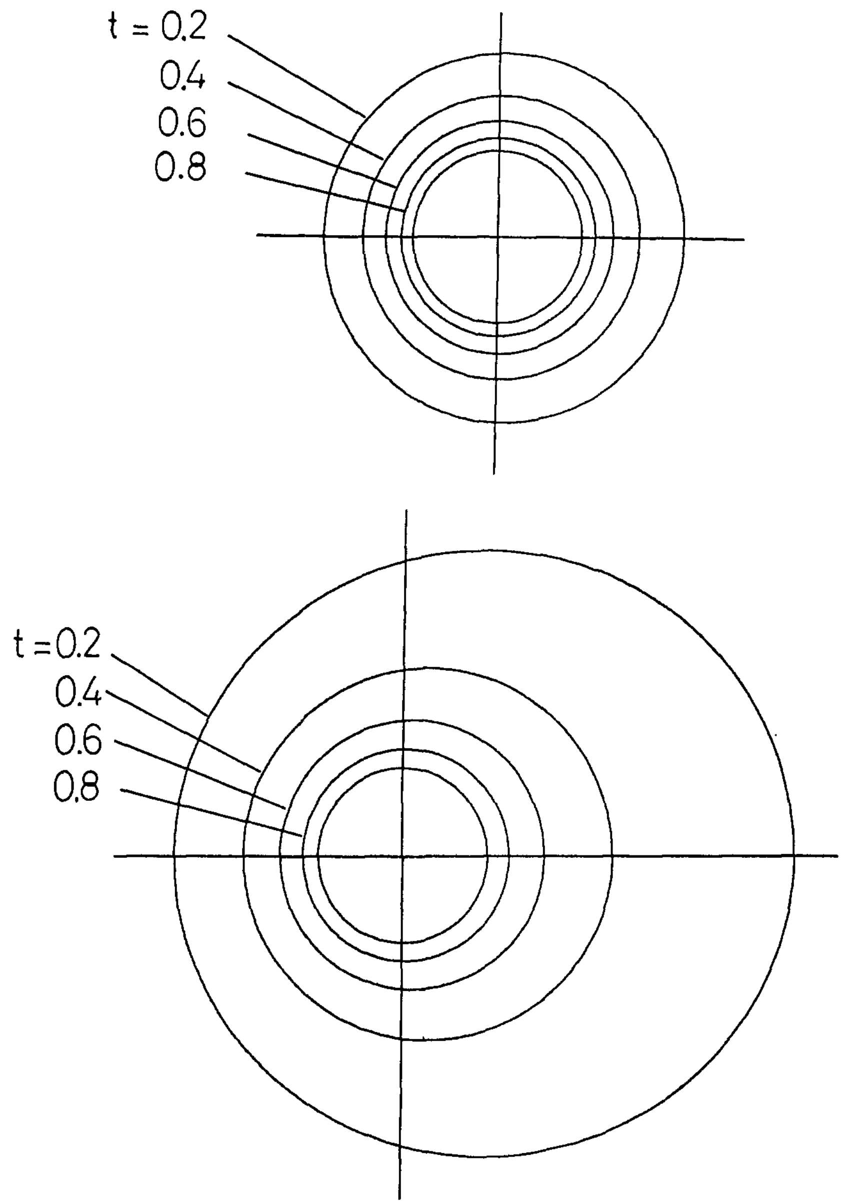

Fig. 1. Isothermal lines for Pe = 0.2; (a) $\tau = 1$, (b) $\tau = 25$, (c) $\tau \to \infty$

the sphere is suddenly changed from T_∞ (temperature of the porous medium) to T_w at time $\tau' = 0$. The non-dimensional equations governing the unsteady natural convection caused by this step change in wall temperature may be written under the Boussinesq approximation as

$$\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2\psi}{\partial r^2} = \cos\theta\frac{\partial T}{\partial\theta} + r\sin\theta\frac{\partial T}{\partial r}, \tag{26}$$

$$\frac{\partial T}{\partial\tau} + \frac{\mathrm{R_a}}{\mathrm{r}^2\sin\theta}\left(\frac{\partial\psi}{\partial\theta}\frac{\partial T}{\partial r} - \frac{\partial\psi}{\partial r}\frac{\partial T}{\partial\theta}\right) = \nabla^2 T, \tag{27}$$

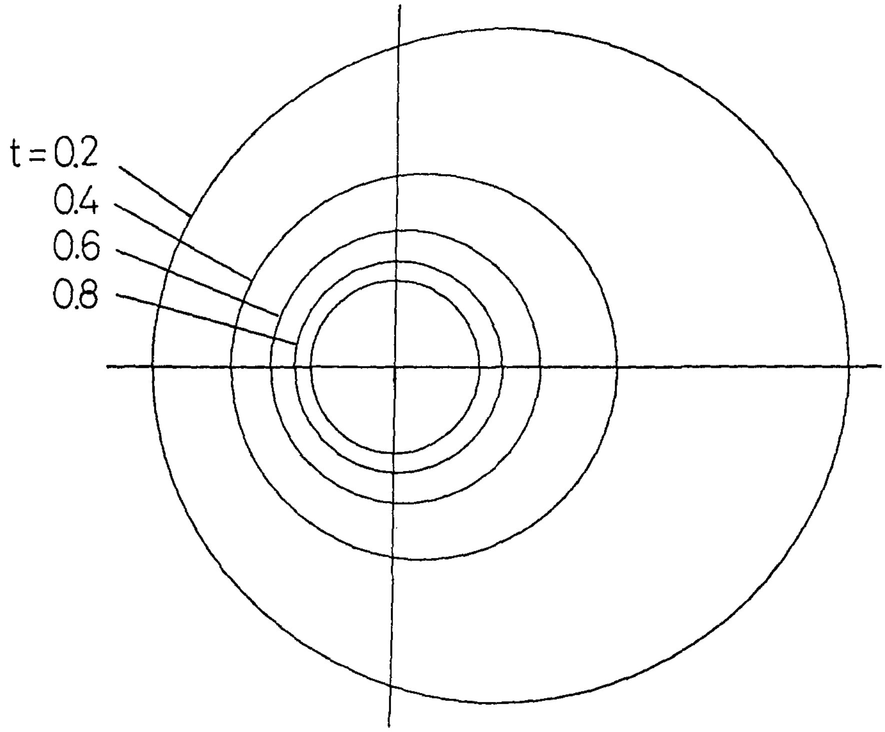

Fig. 1. (*Continued*).

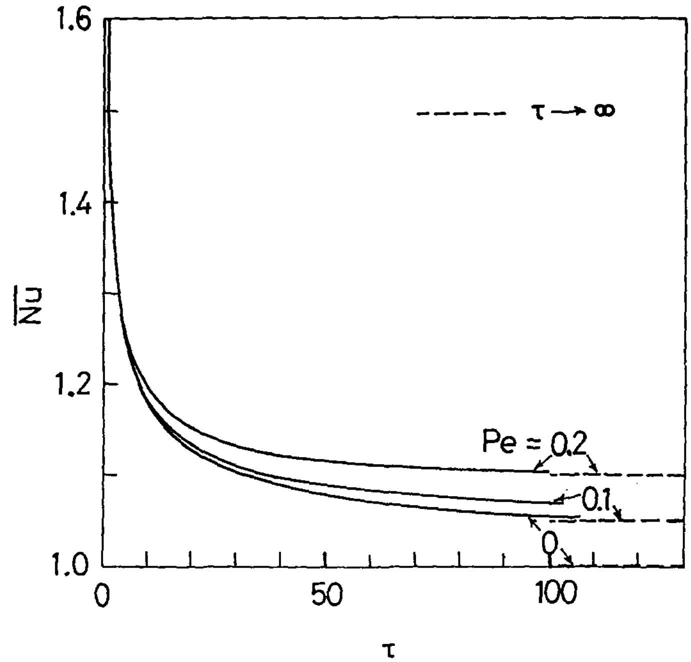

Fig. 2. Timewise variation in $\overline{\mathrm{Nu}}$

where ψ is the stream function related to u_r and u_θ by

$$u_r = \frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta}, \quad u_\theta = -\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}. \tag{28}$$

In the above equations, non-dimensional quantities u_r, u_θ and Ra are defined as

$$u_r = u_r'/U_r, \quad u_\theta = u_\theta'/U_r, \tag{29a}$$

$$\mathrm{Ra} = U_r r_0/\alpha \quad \text{(Rayleigh number)}, \tag{29b}$$

where $U_r = Kg\beta(T_w - T_\infty)/\nu, K, g, \beta$ and ν are the characteristic velocity, the medium permeability, gravity constant, the volumetric coefficient of expansion and the kinematic viscosity of the fluid, respectively. Other non-dimensional variables r, τ and T are defined in the same manner as in (2).

The non-dimensional initial and boundary conditions are

$$\tau < 0 \quad \psi = T = 0, \tag{30a}$$

$$\tau \geqq 0 \quad \psi = 0, T = 1 \quad \text{at } r = 1, \tag{30b}$$

$$\psi/r : \text{finite}, \quad t \to 0 \quad \text{as } r \to \infty. \tag{30c}$$

3.2. Asymptotic solutions for small Ra

We now assume that the Rayleigh number Ra is small and that the solutions may be expanded as

$$\psi = \psi_0(r,\theta,\tau) + \mathrm{Ra}\,\psi_1(r,\theta,\tau) + \cdots, \tag{31}$$

$$T = T_0(r,\theta,\tau) + \mathrm{Ra}\,T_1(r,\theta,\tau) + \cdots. \tag{32}$$

These solutions are uniformly valid for $1 \leqq r \leqq \infty$ and $0 \leqq \tau \leqq \infty$ [13].

The equations for ψ_i and T_i with $i = 0, 1, 2, \ldots$, can be found by substitution of (31) and (32) in (26) and (27). The solutions are obtained only for the first terms T_0 and ψ_0, since the calculation for obtaining the higher-order terms is very cumbersome. The equation for T_0 is the heat conduction equation and its required solution is

$$T_0 = r^{-1}\operatorname{erfc}\eta, \tag{33}$$

where η is defined in (8). The equation for ψ_0 is obtained from (26) in combination with T_0 obtained above and may be written as

$$\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\psi_0}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2\psi_0}{\partial r^2} = -\left(r^{-1}\operatorname{erfc}\eta + \frac{1}{\sqrt{\pi\tau}}\exp(-\eta^2)\right)\sin\theta \tag{34}$$

The required solution of (34) is found to be

$$\psi_0 = \left[\frac{1}{2}(r - r^{-1})\operatorname{erfc}\eta - r^{-1}\left\{\tau\operatorname{erfc}\eta - \frac{2\sqrt{\tau}}{\sqrt{\pi}}(1 - \exp(-\eta^2)) + \frac{2\tau}{\sqrt{\pi}}\eta\exp(-\eta^2) - \tau\right\}\right]\sin^2\theta. \tag{35}$$

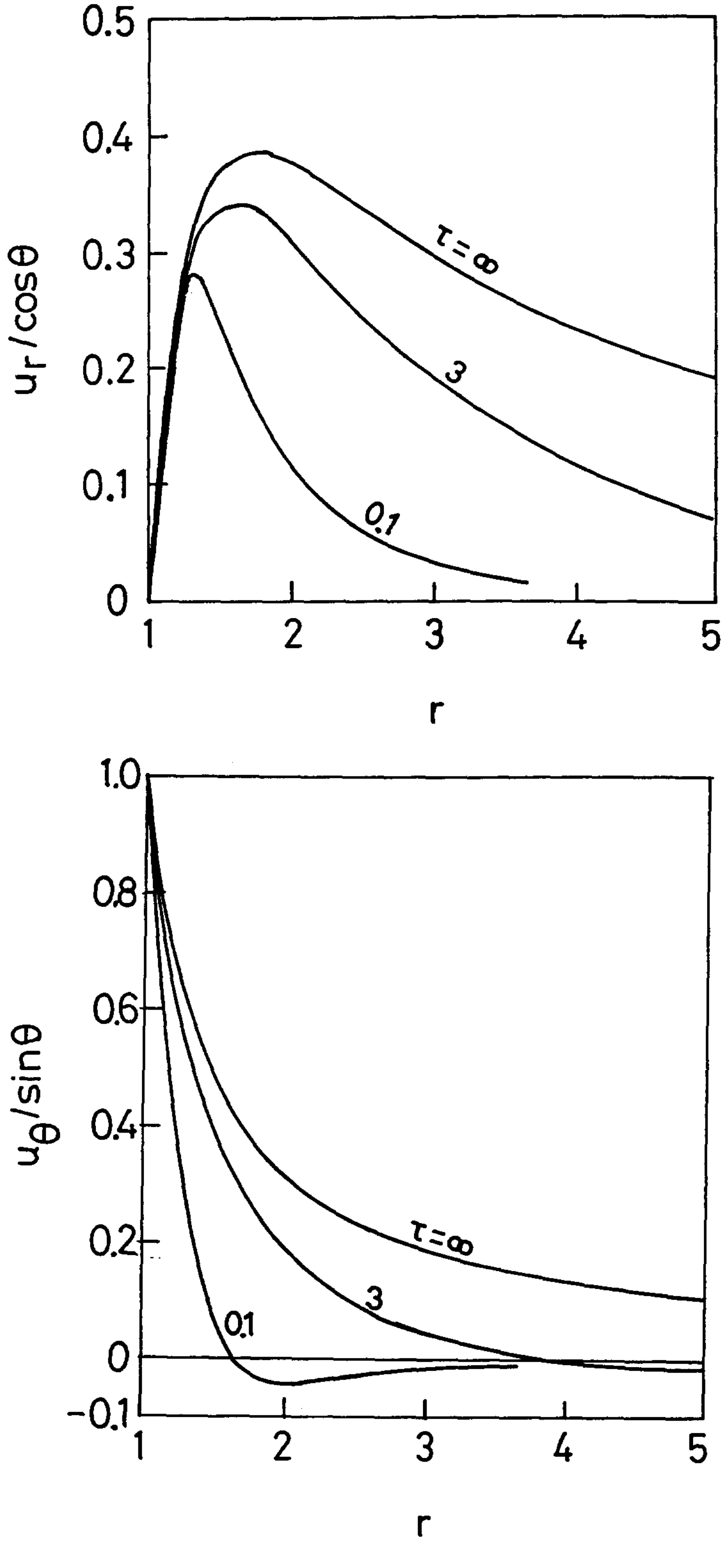

Fig. 3. Velocity distributions; (a) radial velocity, (b) tangential velocity

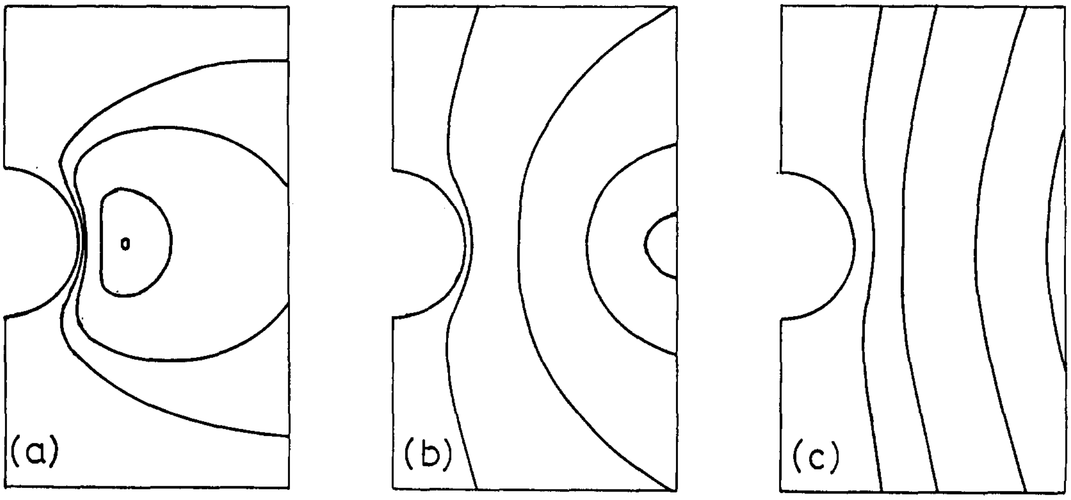

Fig. 4. Transient streamline patterns; (a) $\tau = 0.1$, (b) $\tau = 3$, (c) $\tau \to \infty$

3.3. DISCUSSIONS

Now, we shall show some results for the velocity field obtained from (35). Figs. 3a and 3b show the evolution of $u_r/\cos\theta$ and $u_\theta/\sin\theta$ from stagnancy onwards. It is seen that the tangential velocity u_θ has its maximum at $r = 1$ (the surface of the sphere) and with increasing r decreases to a negative value. This suggests that there exists a vortex ring in the flow as in the case of constant wall heat flux. It is interesting to note that the maximum value of u_θ at $r = 1$ is equal to $\sin\theta$ and does not depend on time.

Fig. 4 shows streamline patterns calculated from (35) for $\tau = 0, 1, 3$ and ∞. The streamlines are symmetric with respect to the plane $\theta = \pi/2$. We can see that a vortex ring is formed around the sphere, whose core is located on the plane $\theta = \pi/2$ and, as time goes on, moves away from the sphere. No appreciable difference in the streamline patterns can be found between the present and the constant wall heat flux case.

The temperature field given in the preceding section is only the first term of the expansion (32), namely, the conduction solution. However, for practical purposes such as, for instance, thermal insulation for heat-generating nuclear waste materials, it is important to know the effect of convection on heat flux from the surface of the sphere. The first convective correction to the temperature field can be found by solving the equation for T_1 which can be written as

$$\frac{\partial T_1}{\partial \tau} - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T_1}{\partial r}\right) - \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T_1}{\partial\theta}\right) = F(r,\tau)\cos\theta, \tag{36}$$

$F(r, \tau)$ being a function of r and τ. Unfortunately, it is difficult to obtain an exact solution of this equation. However, we can estimate, without having to solve (36) the convective effect on heat flux as follows. The solution of (36) satisfying the initial and boundary conditions has the form

$$T_1 = G(r,\tau)\cos\theta, \tag{37}$$

where $G(r, \tau)$ is a function of r and τ. This suggests that the second term T_1 contributes nothing to the mean Nusselt number $\overline{\mathrm{Nu}}$, because of symmetry and that the effect of convection on heat flux is very small and, therefore, porous media are very efficient materials for insulating the hot body thermally, even when natural convection occurs in it. The same result is also found in the steady-state problem considered by Yamamoto. He also found that, in the steady state, the effect of convection on $\overline{\mathrm{Nu}}$ is of the order of Ra^2. We can expect that this is also true for the transient state, suggesting that the mean Nusselt number in the transient state is given by

$$\overline{\mathrm{Nu}} = 1 + \frac{1}{\sqrt{\pi\tau}} + O(\mathrm{Ra}^2). \tag{38}$$

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